Topic: Observer and the airplane

Question: An airplane is flying horizontally at 720 miles/hr, 3 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 10 seconds later?

Answer choices:

A About 400 miles/hr

B About 500 miles/hr

C About 600 miles/hr

D About 700 miles/hr

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Solution: A

To solve for the rate of change of the distance between an observer and an airplane over time, we'll consider the length between the observer on the ground and the plane in the air to be the hypotenuse of a right triangle. The Pythagorean theorem

$$a^2 + b^2 = c^2$$

gives a as the distance between the observer and airplane when the airplane is directly overhead, and b as the distance traveled by the airplane parallel to the ground over the given time period, and c as the diagonal distance from the observer to the airplane after the given period of time.

First you'll need to take the derivative of the Pythagorean theorem with respect to time. Since a is a constant that doesn't change with time, it will become 0 when we differentiate.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We already know from the question that

$$\frac{db}{dt} = 720$$

$$t = 10$$

We're asked to solve for dc/dt, which means that we'll need a few more things first before we can plug into our equation. First we need to find the

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distance for b. We can do this using the speed 720 miles/hr and the time 10 seconds. Convert the time to hours which gives

$$x \text{ hours} = 10 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.00278 \text{ hours}$$

Next find the distance b.

$$b \text{ miles} = 0.00278 \text{ hours} \times \frac{720 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 2 \text{ miles}$$

Now find c using $a^2 + b^2 = c^2$, $a = 3$ and $b = 60$.

$$a^2 + b^2 = c^2$$

$$(3)^2 + (2)^2 = c^2$$

$$9 + 4 = c^2$$

$$13 = c^2$$

$$c \approx 3.61$$

Finally, with $db/dt = 720$, $b = 2$ and $c = 3.61$, we can plug everything into the formula.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(2)(720) = 2(3.61)\frac{dc}{dt}$$

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$$\frac{dc}{dt} \approx 399$$

The rate at which the distance between the person and the plane is increasing is about 399 miles/hr.

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Topic: Observer and the airplane

Question: An airplane is flying horizontally at 540 miles/hr, 4 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 20 seconds later?

Answer choices:

A About 424 miles/hr

B About 324 miles/hr

C About 224 miles/hr

D About 124 miles/hr

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Solution: B

To solve for the rate of change of the distance between an observer and an airplane over time, we'll consider the length between the observer on the ground and the plane in the air to be the hypotenuse of a right triangle. The Pythagorean theorem

$$a^2 + b^2 = c^2$$

gives a as the distance between the observer and airplane when the airplane is directly overhead, and b as the distance traveled by the airplane parallel to the ground over the given time period, and c as the diagonal distance from the observer to the airplane after the given period of time.

First you'll need to take the derivative of the Pythagorean theorem with respect to time. Since a is a constant that doesn't change with time, it will become 0 when we differentiate.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We already know from the question that

$$\frac{db}{dt} = 540$$

$$t = 20$$

We're asked to solve for dc/dt, which means that we'll need a few more things first before we can plug into our equation. First we need to find the

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distance for b. We can do this using the speed 540 miles/hr and the time 20 seconds. Convert the time to hours which gives

$$x \text{ hours} = 20 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.0056 \text{ hours}$$

Next find the distance b.

$$b \text{ miles} = 0.0056 \text{ hours} \times \frac{540 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 3 \text{ miles}$$

Now find c using $a^2 + b^2 = c^2$, $a = 4$ and $b = 3$.

$$a^2 + b^2 = c^2$$

$$(4)^2 + (3)^2 = c^2$$

$$16 + 9 = c^2$$

$$25 = c^2$$

$$c = 5$$

Finally, with $db/dt = 540$, $b = 3$ and $c = 5$, we can plug everything into the formula.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(3)(540) = 2(5)\ \frac{dc}{dt}$$

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$$\frac{dc}{dt} = 324$$

The rate at which the distance between the person and the plane is increasing is about 324 miles/hr.

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Topic: Observer and the airplane

Question: An airplane is flying horizontally at 360 miles/hr, 5 miles above the ground, when it passes an observer on the ground. How fast is the distance between the person and the plane increasing 40 seconds later?

Answer choices:

A 125 miles/hr

B 225 miles/hr

C 325 miles/hr

D 425 miles/hr

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Solution: B

To solve for the rate of change of the distance between an observer and an airplane over time, we'll consider the length between the observer on the ground and the plane in the air to be the hypotenuse of a right triangle. The Pythagorean theorem

$$a^2 + b^2 = c^2$$

gives a as the distance between the observer and airplane when the airplane is directly overhead, and b as the distance traveled by the airplane parallel to the ground over the given time period, and c as the diagonal distance from the observer to the airplane after the given period of time.

First you'll need to take the derivative of the Pythagorean theorem with respect to time. Since a is a constant that doesn't change with time, it will become 0 when we differentiate.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

We already know from the question that

$$\frac{db}{dt} = 360$$

$$t = 40$$

We're asked to solve for dc/dt, which means that we'll need a few more things first before we can plug into our equation. First we need to find the

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distance for b. We can do this using the speed 360 miles/hr and the time 40 seconds. Convert the time to hours which gives

$$x \text{ hours} = 40 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x \text{ hours} = 0.0111 \text{ hours}$$

Next find the distance b.

$$b \text{ miles} = 0.0111 \text{ hours} \times \frac{360 \text{ miles}}{\text{hour}}$$

$$b \text{ miles} \approx 4 \text{ miles}$$

Now find c using $a^2 + b^2 = c^2$, $a = 5$ and $b = 4$.

$$a^2 + b^2 = c^2$$

$$(5)^2 + (4)^2 = c^2$$

$$25 + 16 = c^2$$

$$41 = c^2$$

$$c \approx 6.4$$

Finally, with $db/dt = 360$, $b = 4$ and $c = 6.4$, we can plug everything into the formula.

$$2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(4)(360) = 2(6.4)\frac{dc}{dt}$$

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$$\frac{dc}{dt} = 225$$

The rate at which the distance between the person and the plane is increasing is 225 miles/hr.

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